Linear Algebra-I
Mid-semestral exam
Bmath First Year, AY: 24-25

Time: 2 hours 30 minutes

Maximum marks: 30

Notation: $P_n(\mathbb{R})$ *denotes the real vector space of all real polynomials of degree atmost* n, and A^t denotes the transpose of the matrix A.

Answer all questions. No marks will be awarded in absence of proper justification.

(1) Determine if the set $\mathcal{B} = (1, x^2 - x + 5, 4x^3 - x^2 + 5x, 3x + 2)$ forms a basis for the vector space $P_3(\mathbb{R})$ over $\mathbb{R}$. (5)

(2) For a square matrix A, trace of A is defined as the sum of the diagonal entries of A.
(a) Find the dimension of the vector space of all real $n \times n$ matrices with trace zero.
(b) Find a basis for this space. (3+3)

(3) State and prove *rank-nullity theorem* for a linear transformation. (5)

(4) (a) Define null space of a matrix.
(b) Show that for a real square matrix A, the null space of A is same as the null space of A^tA.
(c) What can you conclude about the range of A and the range of A^tA? Justify. (1+4+3)

(5) Define a linear transformation $T : P_3(\mathbb{R}) \to P_3(\mathbb{R})$ by
$$T(p)(x) = xp'(x) - p(x),$$
where p' denotes the derivative of p.
(a) Find the matrix of T with respect to the standard basis of $P_3(\mathbb{R})$, ie. $(1, x, x^2, x^3)$.
(b) Find bases for the null space and the range space of T. (3+3)

****